RF–DC Differences of Thermal Voltage Converters Arising from Input Connectors

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***Abstract*—The RF–dc differences of thermal voltage converters (TVC's) caused by skin effect and transmission-line effects of different length input structures have been studied. Some discrepancies do exist between simple mathematical models and measured results for commonly used input connectors. This paper reports a study of these discrepancies and some worst-case results of changes in RF–dc difference due to connection and disconnection of TVC's.**

I. Introduction

Measurement errors may occur when different input connector configurations are used for high-frequency voltage measurement. Sometimes it is necessary to account for variations in the input structure, for example, between the test configuration at the time of calibration and at the time of use. It is, therefore, desirable to study the variations and estimate the magnitudes of the differences. Formulas do exist which can be used to calculate the skin effect and transmission-line effect, but the practical situations are generally more complicated. In this study, these effects have been investigated by inserting 50-Ω matched impedance air transmission lines of different lengths between the plane of reference and the thermal voltage converter (TVC).

II. Skin Effect

The RF–dc difference, d, of a TVC may be defined by

$$d = \frac{2V_{RF}}{|V_{+dc}| + |V_{-dc}|} - 1$$

where V_{RF}, V_{+dc}, and V_{-dc} are the magnitudes of the RF, dc plus, and dc minus voltages required to produce the same output from the converter thermoelement. The RF–dc difference contribution caused by the skin effect of the input connector and related input transmission line, e.g., a coaxial tee or adaptor, is positive and increasing with increasing frequency. Sometimes the skin effect contribution is not easily observed, either because it is small in absolute value due to the presence of a large input resistance, or because it is small in a relative sense due to the existence of large RF–dc differences caused by the transmission-line effects. The skin effect can be directly observed in low-voltage range TVC's around 1 MHz. The RF–dc difference contribution (d_s) caused by the skin effect can be calculated by (1):

$$d_s = \frac{(R_{RF} - R_d)}{R_{in}} \tag{1}$$

where R_{RF} and R_d are, respectively, the RF and dc resistance of the input structure, i.e., connector and line, and R_{in} is the input resistance of the TVC as measured from the reference plane. In the frequency range where skin effect is most significant, from about 100 kHz to a few megahertz, a dc measurement of R_{in} is suitable because at these frequencies $Z_L \approx R_{in}$. Z_L is the input impedance of the TVC.

To calculate the RF resistance per unit length, R'_{RF} (in Ω/m) for coaxial structures at 1 MHz and above, Stratton's equation can be used [1]:

$$R'_{RF} \approx 22.2\sqrt{f}\left(\frac{\sqrt{\rho_i \mu_i}}{a} + \frac{\sqrt{\rho_o \mu_o}}{b}\right) \tag{2}$$

where

- a outer diameter of inner conductor in meters,
- b inner diameter of outer conductor in meters,
- f frequency in hertz,
- ρ_i resistivity of inner conductor in ohm meters,
- ρ_o resistivity of outer conductor in ohm meters,
- μ_i magnetic permeability of inner conductor in henry per meter,
- μ_o magnetic permeability of outer conductor in henry per meter.

The calculated results of RF resistance of precision, coaxial, air–dielectric transmission lines at 1 MHz and above are shown in the graphs in [1]. Below 1 MHz, R_{RF} is related to the thickness of the air line as shown in the graph of resistance ratio R_{ac}/R_{dc} in the "Radio Engineer's Handbook" [2] which has been used in the calculations. Equation (2) can still be used to approximate the RF resistance between 100 kHz and 1 MHz. Thus, the RF–dc difference caused by the skin effect of the connector and air line can be obtained from (1), and it is proportional to the length of the connector and line. Above 2 MHz, for the tubular coaxial connectors and lines, the RF resistance is much greater than the dc resistance. The RF–dc difference, d_s, is then approximately proportional to $\sqrt{f}$ and can be expressed as [3]:

$$d_s = A\sqrt{f} \tag{3}$$

where A is relatively constant, and its value may differ between theory and practice, and depends on workmanship and materials.

RF–DC difference changes predicted from (1) for the insertion of a 10- or 20-cm, 50-ohm air line between a TVC and the plane of reference were calculated. There were significant differences between the measured and calculated values and some results for 0.5-V and 1-V TVC's are given in Table I. The standard deviations were less than 3 ppm up to 1 MHz and 5 ppm at 3 MHz. (The experimental data in parentheses in Table I also

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TABLE I
CALCULATED AND MEASURED TRANSFER DIFFERENCES (IN ppm) FOR SKIN EFFECT CONTRIBUTION TO RF–DC DIFFERENCES

TVC Range	Length of Line	100 kHz Cal	100 kHz Meas*	200 kHz Cal	200 kHz Meas	500 kHz Cal	500 kHz Meas	1 MHz Cal	1 MHz Meas	3 MHz Cal	3 MHz Meas
1.0 V 50 Ω	10 cm							63	166 (165)	122	338 (328)
	20 cm							126	208 (207)	244	393 (384)
0.5 V 90 Ω	10 cm	5	12 (12)	11	20 (20)	22	40 (40)	35	73 (71)	68	153 (139)
	20 cm	11	22 (22)	22	45 (45)	44	79 (77)	70	128 (122)	135	266 (209)
1.0 V 200 Ω	10 cm	2	4 (4)	5	9 (9)	10	19 (18)	16	31 (25)	31	66 (16)
	20 cm	5	10 (10)	10	22 (21)	20	37 (33)	32	51 (37)	61	86 (−43)

*Measured values in parentheses include transmission line effect.

TABLE II
VARIATION IN RF–DC DIFFERENCE BETWEEN TWO NOMINALLY IDENTICAL TEES FOR A 30-Ω TVC

Freq.	20 kHz	50 kHz	100 kHz
d (ppm)	7	11	19

include the transmission-line effect which was determined from the 100-MHz values in Table III extrapolated to lower frequencies using the frequency-squared relationship.)

Tests were also made on a 0.3-V TVC with 30-Ω input resistance. Using two type 874 tees with the same length, but with different wall thicknesses and materials, the measured RF–dc differences for this TVC are different even at a few tens of kilohertz. The results are shown in Table II. For another TVC with 90-Ω input resistance and 0.5 V applied, the difference between these two tees was 16 ppm at 500 kHz, and 37 ppm at 1 MHz.

The measured RF–dc differences are larger than the theoretical values. Some possible causes for these differences, which arise from the construction of the connectors and air lines, are variations in conductor surface conditions, purity of conductor material, the presence of some unaccounted-for permeable material, and frequency dependent contact resistance between parts of the air line structure [4].

III. TRANSMISSION-LINE EFFECTS

A. Fundamental Relations

From basic transmission-line equations, the voltage relationships between points p and q in Fig. 1 can be expressed approximately as

$$U_p = U_q \cosh \gamma l + I_q Z_C \sinh \gamma l$$

$$I_q = \frac{U_q}{Z_L}.$$

Therefore

$$U_p = U_q\left(\cosh \gamma l + \frac{Z_C}{Z_L} \sinh \gamma l\right) \tag{4}$$

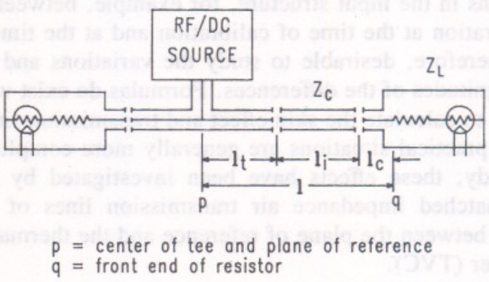

Fig. 1. Connection diagram of two TVC's showing inserted air line.

where

- Z_C characteristic impedance of the line,
- Z_L input impedance of the TVC,
- γ propagation constant of the line,
- l length shown in Fig. 1,
- U_p voltage phasor at the plane of reference in the center of the tee,
- U_q voltage phasor at the front end of the TVC resistor.

This equation is widely used in RF voltage measurements. For a lossless transmission line, with

$$\gamma = j\omega\sqrt{LC} \text{ and } Z_C = \sqrt{\frac{L}{C}}$$

where

- L inductance per unit length of the line,
- C capacitance per unit length of the line,

the voltage ratio becomes

$$\frac{U_q}{U_p} = \frac{1}{\cos(\sqrt{LC}\,\omega l) + j\dfrac{Z_C}{Z_L}\sin(\sqrt{LC}\,\omega l)} \tag{5}$$

The RF–dc differences (d_t) arising from transmission-line effects can be expressed approximately by

$$d_t = \left|\frac{U_p}{U_q}\right| - 1 \approx -\frac{1}{2} LC\omega^2 l^2 \left|1 - \frac{Z_C^2}{Z_L^2}\right|. \tag{6}$$

TABLE III
MEASURED CHANGES IN RF–DC DIFFERENCE(%) RESULTING FROM THE INSERTION OF 10- AND 20-cm AIR LINES

	10 cm			20 cm		
TVC	30 MHz	50 MHz	100 MHz	30 MHz	50 MHz	100 MHz
H 1V 50 Ω		−0.31	−0.88		−0.56	−0.73
A 0.5V 90 Ω	−0.09	−0.30	−1.56	−0.44	−1.36	−6.17
C 0.5V 90 Ω	−0.075	−0.28	−1.52	−0.37	−1.27	−6.14
H 1V 200 Ω	−0.49	−1.37	−5.48	−1.34	−3.71	−14.2
C 2V 360 Ω	−0.51	−1.46	−5.73	−1.43	−4.06	−15.9
C 12.5V 2.7 kΩ	−0.52	−1.44	−5.68	−1.43	−3.88	
A 5V 1 kΩ	−0.77	−2.14	−8.61	−1.94	−5.42	−21.8
C 50 V 5 kΩ	−1.01	−2.83	−10.83	−2.44	−6.64	−23.67

TABLE IV
CALCULATED CHANGE IN RF–DC DIFFERENCES RESULTING FROM THE INSERTION OF 10- AND 20-cm AIR LINES

l_i(cm)	l(cm)	d	Δd	Δd 30 MHz	Δd 50 MHz	Δd 100 MHz
0	7.4	$-1.18 \times 10^{-18} f^2$	0	0	0	0
10	17.4	$-6.59 \times 10^{-18} f^2$	$-5.41 \times 10^{-18} f^2$	−0.49%	−1.35%	−5.41%
20	27.4	$-16.4 \times 10^{-18} f^2$	$-15.22 \times 10^{-18} f^2$	−1.37%	−3.81%	−15.22%

d is the transmission line expression for RF–dc difference.
Δd is the change in RF–dc difference due to air line insertion.
l and l_i are as shown in Fig. 1.

B. The Influence of Z_L

Equation (6) shows that, for the same connector, the RF–dc difference is a function of the input impedance of the TVC. For simply constructed TVC's, the change in RF–dc difference for the same change in length of the input line increases as the input impedance of the TVC increases. This trend continues with increasing input impedance until the impedance is larger than about 200 Ω. Above this value, the effect due to changes in input line length remains relatively constant. Table III shows this relationship going from C-0.5V to the group of TVC's labeled H-1V, C-2V, and C-12.5V. TVC's A-5V and C-50V, which have internal shields and larger input capacitances, will be discussed later.

Input impedance, Z_L, is a function of frequency and the construction of the TVC. For TVC's with no internal shield and having range resistors of 200–400 Ω, the input impedances have been measured from 1 to 100 MHz and found to be slowly decreasing. This is in contrast to theoretical calculations given in [5] for simple resistors which show Z_L increasing with frequency for values below 300 Ω. However, measurements on a particular TVC with a complicated internal structure gave a ratio of dc resistance to Z_L at 100 MHz of nearly 10:1. Determination of the value of Z_L may, therefore, require a direct measurement.

C. Calculation of Transmission-Line Effect

When $(Z_L)^2 >> (Z_C)^2$, the first-order approximation for the RF–dc difference contribution from the transmission line is

$$d_t = -\tfrac{1}{2} LCl^2\omega^2 = -Bl^2f^2$$

where B is $2\pi^2 LC$. Therefore the changes in RF–dc differences caused by the transmission-line effect are proportional to l^2 and f^2. In Table III it can be seen that Δd is approximately proportional to f^2 when input resistance is 200 Ω or greater. The skin effect can usually be neglected for these input resistance ranges. For a coaxial line: $L = 2 \times 10^{-7} \ln(b/a)$ in henries per meter, $C = 1/1.8 \times 10^{10} \ln(b/a)$ in farads per meter,

$$d = 2.19 \times 10^{-20} l^2 f^2 \text{ (with } l \text{ in cm)} \qquad (7)$$

Quantities a and b are the same as in (2). The calculated results are shown in Table IV.

D. Determination of Equivalent Line Length

As shown in Tables III and IV, the theoretical values for Δd are close to, but slightly less than, the experimental results for the 1-V and 12.5-V ranges. One way to describe the difference between theoretical and measured values is in terms of an equivalent electrical length which is longer than the physical length.

The equivalent electrical length was calculated by relating the measured changes in the RF–dc difference when two air lines of known length (l_1 = 10 cm and l_2 = 20 cm) were inserted between the TVC and the plane of reference. Using the notation in Fig. 1,

$$l = l_0 = l_t + l_c \qquad d_{l0} = -Bl_0^2 f^2 + d_0(f)$$

$$l = l_0 + l_1 \qquad d_{l1} = -B(l_0 + l_1)^2 f^2 + d_0(f)$$

$$l = l_0 + l_2 \qquad d_{l2} = -B(l_0 + l_2)^2 f^2 + d_0(f)$$

where $d_0(f)$ is the RF–dc difference of the TVC with $l = 0$

$$\Delta d_1 = d_{l1} - d_{l0} = -B(l_1^2 + 2l_1 l_0) f^2$$

$$\Delta d_2 = d_{l2} - d_{l0} = -B(l_2^2 + 2l_2 l_0) f^2.$$

Therefore

$$\frac{\Delta d_2}{\Delta d_1} = \frac{l_2^2 + 2l_2 l_0}{l_1^2 + 2l_1 l_0} = m \qquad l_0 = \frac{l_2^2 - ml_1^2}{2(ml_1 - l_2)}. \tag{8}$$

The quantities l_1, d_1, l_2, d_2 were measured and l_0 was calculated from (8). The method of inserting first the 10 cm and then the 20 cm lines allows the user to determine the effective electrical length. Note that the B term which contains LC drops out. For TVC's constructed of only a resistor and a thermoelement, the values for l_0 obtained this way were about 2 cm longer than the physical lengths. Two of the TVC's studied, A-5V and C-50V, contained shield structures inside the outer grounded cylinder. For these converters, l_0 was found to be longer by as much as 6–11 cm than the physical length. The results for a A-5V and C-50V given in Table III show this longer apparent length. For some shield structures, an effect like that of an internal transmission line may produce voltages even higher than those at the input connector. The observations that the physically measured lengths and the experimentally determined values are different may be described in terms of equivalent electrical lengths, which are longer than the physical lengths. These measurements show that to calculate the effect on RF–dc difference caused by varying the input line length by using connector adapters or otherwise moving the plane of reference requires knowledge of the equivalent l_0.

IV. Combined Results

The combined effects of skin effect and the transmission line have been studied, in part, by the fitting of data to equations of the form

$$d = d_s + d_t = A\sqrt{f} \quad Bl^2 f^2$$

containing square root of frequency and frequency squared as described in [3]. Some results showing both contributions to RF–dc difference are given in Fig. 2. The plot shows the change in RF–dc difference resulting from the insertion of a 10-cm air line. The positive slope at the lower frequencies is due to the skin effect contribution, and the negative slope at higher frequencies results from the transmission-line effect.

V. Influence of the Impedance of the Reference TVC

Generally, the input impedance of the reference TVC has no significant effect on the test TVC and its input transmission line characteristics. However, experiments made for this study do show a small effect at frequencies around 70–100 MHz. The results are shown in Table V. Reflected signals are a likely cause for this variation.

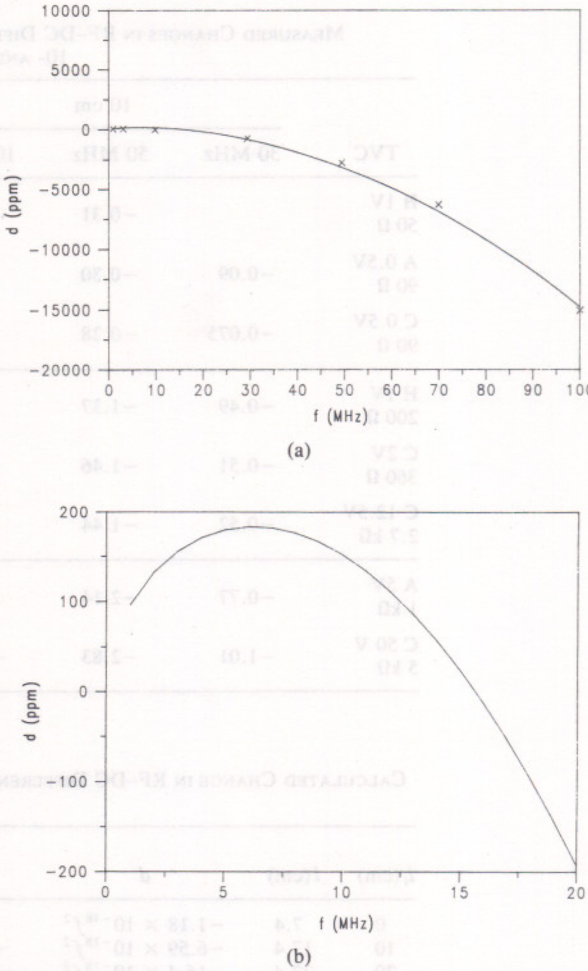

Fig. 2. Change in RF–dc difference for a 0.5-V TVC resulting from the insertion of a 10-cm air line. Data points represented by X's. Continuous curve generated by nonlinear fit to all data given in Fig. 2(a)–(b). Fig. 2(b) is the low frequency part of the whole curve.

VI. Repeatability of Specific Connectors

During the calibration and measurement process, different tees and different locations on the same tee may be reconnected several times. Previously, tests were made on the repeatability of RF–dc difference after careful disconnection and reconnection [6]. Some additional data regarding the repeatability of such reconnections have been taken as part of this study. Although by no means exhaustive, the results indicate how large the worst case changes may be.

Each time a connection is made, there may be small differences of position and contact characteristics. Three type 874 tees and one type BNC tee which had been in use for many years were randomly chosen and connected without cleaning. The results, with standard deviations of about 10 ppm, were measured in three trials and are shown in Fig. 3. Generally the range of variation increases with the measurement frequency and age of the connector, and decreases with the input resistance of the TVC.

TABLE V
INFLUENCE OF THE REFERENCE TVC IMPEDANCE

Length of Air Line Inserted	Test TVC	Reference TVC	30 MHz	50 MHz	70 MHz	100 MHz
10 cm	H 1V 200 Ω	C 0.5V 90 Ω	−0.466%	−1.326%	−2.607%	−5.277%
		C 1V 184 Ω	−0.485%	−1.366%	−2.684%	−5.476%
10 cm	C 12.5V 2.7 kΩ	Pt 10V 1 kΩ	−0.515%	−1.438%		−5.677%
		C 20V 2 kΩ	−0.510%	−1.430%		−5.744%
20 cm	H 1V 200 Ω	C 0.5V 90 Ω	−1.341%	−3.724%	−7.210%	−14.17%
		C 1V 184 Ω	−1.342%	−3.711%	−7.236%	−14.24%

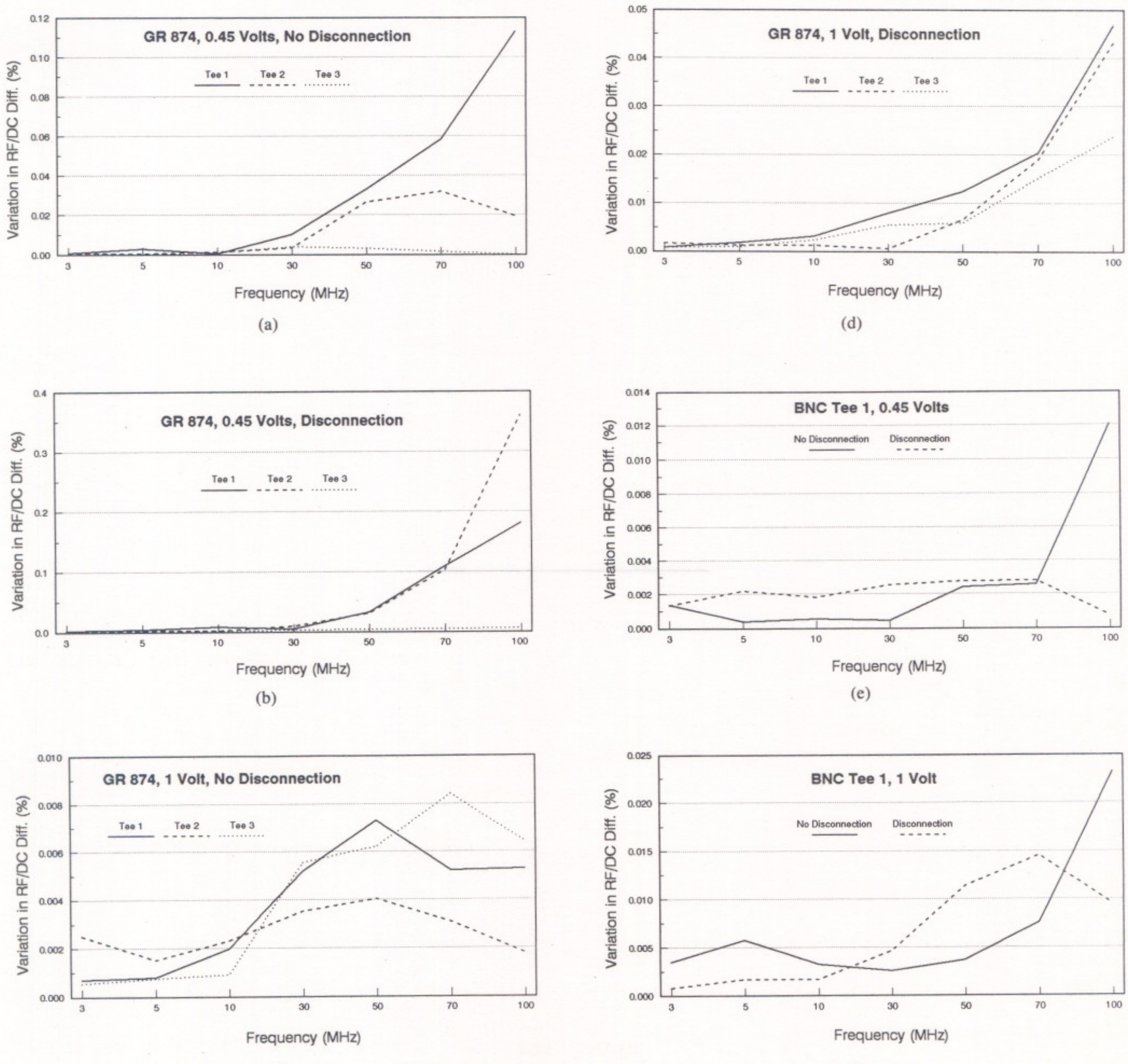

Fig. 3. Worst case variations in RF-dc differences caused by connection and disconnection. Plots (a)–(d) give data for type 874 connectors and tees. Plots (e) and (f) show a limited sample of data for type BNC connectors and tees.

VII. Conclusion

As predicted from theoretical calculations, skin effect in the input connection structure has been observed to be significant for TVC's with lower input resistances at frequencies around 1 MHz. However, experimental results differ from calculated values under some conditions. When the input resistance of the TVC is larger and the frequency is higher, the transmission-line effect is more important. The experimental results differ from calculated values for this effect due to differences between physical and apparent electrical lengths. For high-accuracy measurements, it is highly desirable to reduce the differences in the lengths and in variations of connector types as much as practical. Standardized dimensions and lengths for tees would be desirable for various connector types and RF probes.

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